

Time: 3 hours

Marks: 80

**N.B.: (1) Question No 1 is compulsory****(2) Attempt any three questions out of the remaining five questions****(3) Non Programmable calculator is allowed****Q.1)**

- a) Find the value of the integral  $\int_C (x + y)dx + x^2ydy$  along  $y=x^2$  having (0,0) and (3,9) end points . [5]
- b) Find the Fourier Series representing by  $f(x)=x$ ,  $0 < x < 2\pi$ . [5]
- c) Find the Fourier transforms of  $f(x)=1-x^2$ ,  $|x| < 1$ ;  $0$  for  $|x| > 1$  [5]
- d) Prove that  $\left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{E(e^x)}{\Delta^2 e^x} = e^x$  the interval of differencing being  $h$ . [5]

**Q.2)**

- a) Prove that  $\frac{1}{2} - x = \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{2n\pi x}{l}$ ,  $0 < x < l$  [6]
- b) Using the method of Lagrange's multipliers solve the following NLPP  
Optimize  $z=4x_1+8x_2-x_1^2-x_2^2$  subject to  $x_1+x_2=4$ ,  $x_1, x_2 \geq 0$  [6]
- c) Expand  $f(z) = \frac{1}{z^2(z-1)(z+1)}$  about  $z=0$  indicating the region of convergence. [8]

**Q.3)**

- a) If  $f(1)=4$ ,  $f(2)=4$ ,  $f(7)=5$  and  $f(8)=4$ , find  $f(5)$  using Lagrange's Interpolation formula [6]
- b) The vibrations of an elastic string is governed by the partial differential equation  $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ . The length of the string is  $\pi$  and the ends are fixed. The initial velocity is zero and the initial deflection is  $u(x,0)=2(\sin x + \sin 3x)$ . Find the deflection  $u(x,t)$  of the vibrating string for  $t > 0$ . [6]
- c) Use the Kuhn Tucker conditions to solve the following NLPP [8]  
Maximize  $z=2x_1+3x_2-x_1^2-2x_2^2$   
subject to  $x_1+3x_2 \leq 6$ ,  
 $5x_1+2x_2 \leq 10$ ,  $x_1, x_2 \geq 0$

**Q4)**

- a) Prove that  $x(\pi - x) = \frac{\pi^2}{6} - \left[ \frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \dots \dots \right]$  in the interval  $(0, \pi)$  [6]
- b) Evaluate the integral using Cauchy's Integral formula  $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$  where  $C$  is the circle  $|z| = \frac{3}{2}$  [6]
- c) Find the solution of one dimensional heat flow is given by  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$  for which  $u(0, t) = u(l, t)$ ,  $u(x, 0) = \sin \frac{\pi x}{l}$ . [8]

Q5)

- a) If  $y_0 = -8, y_1 = -6, y_2 = 22, y_3 = 148, y_4 = 492, y_5 = 122$  find  $y_6$ . [6]
- b) Find the complex form of the Fourier series for  $f(x) = e^{ax}$  over the interval  $(-l, l)$  [6]
- c) The diameter of a semicircular plate of radius  $a$  is kept at  $0^\circ\text{C}$  and the temperature at the semicircular boundary is  $T^\circ\text{C}$ . Find the steady state temperature in the plate. [8]

Q6)

- a) Using method of separation of variable solve  $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$  where  $u(x, 0) = 6e^{-3x}$  [6]
- b) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ . [6]
- c) (i) Use appropriate difference formula to find  $t$  when  $p=84$  from the following data [4]

p	60	70	80	90
t	226	250	276	304

- (ii) Express  $f(x)=x^4-12x^3+42x^2-30x+9$  in terms of factorial polynomials. [4]