

(3 hours)

**Total Marks: 80**

**N.B:** (1) Question no.1 is **compulsory**.

(2) Attempt any **three** questions from remaining **five** questions.

(3) **Figures** to the **right** indicate **full** marks.

(4) Assume suitable data if necessary.

1. (a) Find the Laplace Transform of  $(t^2 \sinh t)^2$ . (5)

(b) If  $A = \begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix}$ , find  $2A^3 - A^2 - 35A - 44I$ . (5)

(c) Evaluate  $\int_0^{2+i} (\bar{z})^2 dz$  along the curve  $x = 2y^2$ . (5)

(d) In a Binomial distribution consisting of 5 independent trials, the probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter 'p' of the distribution. (5)

2. (a) Find an analytic function whose real part is  $(x^3 - 3xy^2 + 3x^2 - 3y^2 + 1)$ . (6)

(b) Evaluate  $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$ . (6)

(c) Find the eigenvalues and bases for eigenspaces of the following matrix

$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ . (8)

3. (a) Find the inverse Laplace Transform of  $\frac{2s+3}{(s+2)(s+1)^2}$ . (6)

(b) Show that the matrix A is diagonalisable, where  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ . Also find the transforming and the diagonal matrix. (6)

(c) Using Kuhn-Tucker conditions solve the following NLPP  
 Maximise  $Z = 2x_1 + x_2 - x_1^2$   
 subject to  $2x_1 + 3x_2 \leq 6; 2x_1 + x_2 \leq 4; x_1, x_2 \geq 0$ . (8)

4.(a) Find the bilinear transformation which maps the points  $z = 1, i, -1$  onto the points  $w = i, 0, -i$ . (6)

(b) Find the orthogonal trajectory of the family of curves given by  $3x^2y + 2x^2 - y^3 - 2y^2 = c$  (6)

(c) Solve the following NLPP using Lagrange's multipliers method (8)

Optimise  $Z = -10x_1 - 6x_2 - 4x_3 + x_1^2 + x_2^2 + x_3^2$   
 subject to  $x_1 + x_2 + x_3 = 7; x_1, x_2, x_3 \geq 0$ .

5.(a) Show that the matrix A is derogatory where  $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ . (6)

(b) Evaluate  $\oint_C \frac{z^2}{(z-1)^2(z-2)} dz$  where C is the circle  $|z| = 2.5$ . (6)

(c) Find the inverse Laplace Transform of  $\frac{1}{(s-1)(s^2+4)}$  using convolution theorem. (8)

6. (a) Using the residue theorem evaluate  $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ . (6)

(b) From the following data calculate Spearman's rank correlation coefficient between X and Y

X: 60 62 64 66 68 70 72 74

Y: 92 83 101 110 128 119 137 146

(6)

(c) Reduce the following quadratic form to canonical form. Also find its rank and signature

$2x^2 + y^2 - 3z^2 + 12xy - 8yz - 4zx$ . (8)