Paper / Subject Code: 58501 / Applied Mathematics - I.

1T01811 - F.E.(Sem I)(ALL BRANCHES) (CBSGS) / 58501 - Applied Mathematics - I.

(3 hours) **Total marks: 80**

N.B.: (1) Question **No. 1** is **compulsory**

(2) Attempt **any Three** from remaining

Q1 a) If $\log \tan x = y$ then prove that $\sinh ny = \frac{1}{2}[\tan^n x - \cot^n x]$ [3]

b) If $u = x^2 y + e^{xy^2}$ Find $\frac{\partial^2 u}{\partial x \partial y}$ [3]

c) If $x = u - uv$, $y = uv - uvw$, $z = uvw$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ [3]

d) Using Maclaurin's series , Prove $e^{e^x} = e + ex + ex^2 + \cdots$ [3]

e) Show that $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary [4]
if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$

f) Find n^{th} derivative of $\frac{x}{(x-1)(x-2)(x-3)}$ [4]

Q2 a) Solve $x^5 = 1 + i$ and find the continued product of the roots. [6]

b) Reduce the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ to the normal form [6]
and find its Rank

c) State and Prove Euler's theorem for two variables hence [8]
find value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where $u = \frac{\sqrt{xy}}{\sqrt{x} + \sqrt{y}}$

Q3 a) Test the consistency of [6]

$2x - y - z = 2$, $x + 2y + z = 2$, $4x - 7y - 5z = 2$

And Solve if consistent.

b) Examine the function for its extreme values [6]

$f(x, y) = y^2 + 4xy + 3x^2 + x^3$

c) If $\sin(\theta + i\varphi) = e^{i\alpha}$ then Prove $\cos^4\theta = \sin^2\alpha = \sinh^4\varphi$ [8]

Q4 a) If $x = u\cos v$, $y = u\sin v$ then [6]
Prove $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$

b) If $\log(x + iy) = e^p(\cos q + i\sin q)$ then [6]
prove that $y = x\tan\left(\tan q \;.\log\sqrt{x^2+y^2}\right)$

c) Solve by Gauss Elimination method [8]

$2x + 3y + 4z = 11$, $x + 5y + 7z = 1$, $3x + 11y + 13z = 25$

Q5 a) Prove $cos^6\,\theta + \sin^6\theta = \frac{1}{8}[3\cos 4\theta + 5]$ [6]

b) Evaluate $\lim_{x \to 0}\left[\frac{1}{x^2} - \cot^2 x\right]$ [6]

c) If $y = \cos(m\sin^{-1} x)$ then [8]

prove that $(1 - x^2)y_{n+2} - (2n+1)x\ y_{n+1} + (m^2 - n^2)y_n = 0$

Q6 a) Check if the following vectors [6]

$X_1 = [1\,,0\,,2\,,1]$, $X_2 = [3\,,1\,,2\,,1]\,,X_3 = [4,6\,,2\,,-4]$,

$X_4 = [-6,0\,,-3\,,-4]$ are linear dependent hence find the relation between them if any.

b) If $f(xy^2, z - 2x) = 0$ then [6]

prove that $2x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 4x$

c) Fit a second degree parabola $y = ax^2 + bx + c$ to the following data [8]

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9