Q. P. Code: 25565

(3hours) [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) Find Laplace transform of $f(t) = \int_0^t u\, e^{-3u}\, sin\, u du.$ 5

b) Show that the set of functions $\{cos\, nx\, , n = 1,2,3 \ldots\}$ is orthogonal on $(0,2\pi)$. 5

c) Does there exist an analytic function whose real part is $u = k(1 + cos\,\theta)$? Give justification. 5

d) The equations of lines of regression are $x + 6y = 6$ and $3x + 2y = 10$. Find i) means of x and y, ii) coefficient of correlation between x and y. 5

2. a) Evaluate $\displaystyle\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt.$ 6

b) Find the image of the triangle bounded by lines $x = 0, y = 0, x + y = 1$ in the z-plane under the transformation $w = e^{i\pi/4} z$. 6

c) Obtain Fourier series of $f(x) = x^2$ in $(0,2\pi)$. Hence, deduce that – 8

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + + \cdots .$$

3. a) Find the inverse Laplace transform of $F(s) = \frac{s}{(s^2+4)^2}$. 6

b) Solve $\frac{\partial^2 u}{\partial x^2} - 100 \frac{\partial u}{\partial t} = 0$, with $u(0,t) = 0, u(1,t) = 0, u(x,0) = x(1-x)$

taking $h = 0.1$ for three time steps up to $t = 1.5$ by Bender –Schmidt method. 6

c) Using Residue theorem, evaluate

i) $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 - 4\cos\theta}$ ii) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{\left(x^2+1\right)^2}$ 8

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4. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0,$

$u(0,t) = 0,\ u(5,t) = 100,\ u(x,0) = 20$ taking $h = 1$ for one-time step. 6

b) Obtain the Taylor's and Laurent series which represent the function

$f(z) = \frac{z-1}{z^2-2z-3}$ in the regions, i) $|z| < 1$ ii) $1 < |z| < 3$ 6

c) Solve $(D^2 + 4D + 8)y = 1$ with $y(0) = 0$ and $y'(0) = 1$ where $D \equiv \frac{d}{dt}$ 8

5. a) Find an analytic function $f(z) = u + iv$, if 6

$$u = e^{-x}\{(x^2 - y^2)\cos y + 2xy\sin y\}$$

b) Find the Laplace transform of

$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ and $f(t+2) = f(t)$ for $t > 0$. 6

c) Obtain half range Fourier cosine series of $f(x) = x, 0 < x < 2$. Using Parseval's identity, deduce that – 8

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots.$$

6. a) If $f(a) = \int_C \frac{4z^2+z+4}{z-a} dz$ where C is the ellipse $4x^2 + 9y^2 = 36$.

Find, i) $f(4)$ ii) $f'(-1)$ and iii) $f''(-i)$ 6

b) Use least square regression to fit a straight line to the following data, 6

x	5	10	15	20	25	30	35	40	45	50
y	17	24	31	33	37	37	40	40	42	41

c) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in form $y = a sin(\pi x / l)$ from which it is released at a time $t = 0$. If the vibrations of a string is given by $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, show that the displacement of a point at a distance x from one end at time t is given by $y(x,t) = a\, sin(\pi x / l)\, cos(\pi ct / l)$. 8