Paper / Subject Code: 49801 / APPLIED MATHEMATICS- III

Time: 3 Hours **Marks: 80**

N.B.:1) Question no.1 is compulsory.
2) Attempt any three questions from Q.2to Q.6.
3) Figures to the right indicate full marks.

Q1. a) Find the Laplace transform of $e^{-t}t\cosh 2t$. **[5]**

b) Find the half-range cosine series for $f(x)=\begin{cases}1 & ,o<x<\frac{a}{2}\\ -1 & ,\frac{a}{2}<x<a\end{cases}$ **[5]**

c) Find $\nabla\left(\bar{a}.\nabla\frac{1}{r}\right)$ where $\bar{a}$ is a constant vector. **[5]**

d) Show that the function $f(z)=z^3$ is analytic and find $f'(z)$ in terms of z. **.[5]**

Q2. a) Find the inverse Z-transform of $F(z)=\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}$,$3<z<4$. **[6]**

b) Find the analytic function whose imaginary part is $\tan^{-1}\left(\frac{y}{x}\right)$. **[6]**

c) Obtain Fourier series for the function $f(x)=\begin{cases}\frac{\pi}{2}+x & ,-\pi<x<0\\ \frac{\pi}{2}-x & ,0<x<\pi\end{cases}$, **[8]**

Hence ,deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ and $\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots$

Q3. a) Find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ using convolution theorem. **[6]**

b) Show that the set of functions $\phi_n(x)=\sin\left(\frac{n\pi x}{l}\right), n=1,2,3\ldots$ is orthogonal in $[0\,,l]$. **[6]**

c) Using Green's theorem evaluate $\oint_C\left(e^{x^2}-xy\right)dx-(y^2-ax)dy$ where C is the circle $x^2+y^2=a^2$. **[8]**

Q4. a) Find Laplace transform of $f(t)=\begin{cases}\frac{t}{a} & ,o<t\leq a\\ \frac{(2a-t)}{a} & ,a<t<2a\end{cases}$ and $f(t)=f(t+2a)$. **[6]**

b) Prove that a vector field $\bar{f}$ is irrotational and hence find its scalar potential $\bar{f}=(y\sin z-\sin x)\,i+(x\sin z+2yz)j+(xy\cos z+y^2)k$. **[6]**

c) Obtain the Fourier expansion of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $0\leq x\leq 2\pi$ and $f(x+2\pi)=f(x)$ .Also deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ **[8]**

Q5.a) Use Gauss's Divergence Theorem to evaluate $\iint_S \bar{N}.\bar{F}ds$ where $\bar{F}=4xi+3yj-2zk$ and S is the surface bounded by x=0, y=0, z=0 and 2x+2y+z=4. **[6]**

b) Find the Z-transform of $f(k)=ke^{-ak}$, k≥ 0 . **[6]**

c) i) Find $L^{-1}\left[\frac{s+2}{s^2(s+3)}\right]$. **[8]**

ii) Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$.

Q6.a) Solve using Laplace transform $(D^2+3D+2)y=2(t^2+t+1)$, with $y(0)=2$ and $y'(0)=0$. **[6]**

b) Find the bilinear transformation which maps the points Z=1, i, -1 onto the points W=i, 0, -i . **[6]**

c) Find Fourier sine integral of $f(x)=\begin{cases}x & ,0<x<1\\ 2-x & ,1<x<2\\ 0 & ,x>2\end{cases}$ **[8]**
