S.E.(ELECTRONICS & TELE-COMMN)(Sem III) (Choice Based) / 51201 - APPLIED MATHEMATICS-III

(3 Hours) **[Total Marks: 80]**

N.B. : 1) Question No. **1** is **Compulsory**.

2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Evaluate $\int_0^\infty e^{-2t} t \sin t \, dt$. (5)

(b) Find a, b, c, d, e if $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2) + i(dx^2 y - 2y^3 + exy)$ is analytic. (5)

(c) Find half range sine series of $f(x) = x(\pi - x), 0 < x < \pi$. (5)

(d) Find directional derivative of $\phi = 4xz^2 + x^2yz$, at $(1,-2,-1)$ in direction of $2i - j - 2k$. (5)

Q.2 (a) Prove that $\nabla r^n = n r^{(n-2)} \bar{r}$. (6)

(b) Find Bilinear Transformation which maps the points $z = -1, 0, 1$ onto the points $w = -1, -i, 1$. (6)

(c) Find i) $L^{-1}\left[\dfrac{e^{-2s}}{s^2 + 3s + 2}\right]$ ii) $L^{-1}\left[\log\left(\dfrac{s^2 + 4}{s + 4}\right)\right]$. (8)

Q.3 (a) Use Gauss's Divergence Theorem to evaluate $\iint_s \bar{N}.\bar{F} \, ds$ where $\bar{F} = 4xi - 2y^2 j + z^2 k$ and s is region bounded by $x^2 + y^2 = 4, z = 0, z = 4$. (6)

(b) Find Laplace Transform of $e^{-2t} \int_0^t u e^{3u} \cos 4u \, du$. (6)

(c) Obtain Fourier series of $f(x) = \begin{cases} x + \dfrac{\pi}{2} & -\pi < x < 0 \\ \dfrac{\pi}{2} - x & 0 < x < \pi \end{cases}$. (8)

Hence deduce $\dfrac{\pi^4}{96} = \dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + \cdots$

Q.4 (a) Show that set of functions $\left\{\frac{\sin x}{\sqrt{\pi}}, \frac{\sin 2x}{\sqrt{\pi}}, \frac{\sin 3x}{\sqrt{\pi}} \cdots\right\}$ form an orthonormal set in $(-\pi, \pi)$. (6)

(b) Find orthogonal trajectories of the family of curves $e^x \cos y - xy = c$. (6)

(c) Prove that $\overline{F} = (6xy^2 - 2z^3)i + (6x^2y + 2yz)j + (y^2 - 6z^2x)k$ is irrotational. Find scalar potential of $\overline{F}$. Hence find the work done of moving particle from $(1,0,2)$ to $(0,1,1)$. (8)

Q.5 (a) Find Fourier Integral representation for $f(x) = \begin{cases} 1 - x^2 & |x| \le 1 \\ 0 & |x| > 1 \end{cases}$. (6)

(b) Solve using Laplace Transform $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = t\,e^{-t}$ given $y(0) = 4$ and $y'(0) = 2$. (6)

(c) Verify Green's Theorem for $\overline{F} = x^2 i - xy\, j$ and c is triangle having vertices $A(0,2)$, $B(2,0)$, $C(4,2)$. (8)

Q.6 (a) Using Convolution theorem, find Inverse Laplace of $\frac{s}{(s^2+4)^2}$. (6)

(b) Prove that $J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3}{x}\sin x + \frac{(3 - x^2)}{x^2}\cos x\right]$. (6)

(c) Find Fourier series for $f(x) = (\pi - x)^2$ in $0 \le x \le 2\pi$. Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ (8)
