

Duration: 3 Hours

Total Marks: 80

N.B. : 1) Q.1. is compulsory.

2) Attempt any three from the remaining.

Q.1. a) Show that the set  $\{e^x, xe^x, x^2 e^x\}$  is linearly independent in  $C^2(-\infty, \infty)$ . (5)

b) Show that  $\int_C \log z dz = 2\pi i$ , where C is the unit circle in the z-plane. (5)

c) Find the projection of  $u=(3,1,3)$  along and perpendicular to  $v=(4,-2,2)$  (5)

d) Find the extremal of  $\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x) dx$  (5)

Q.2. a) If  $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$ , find  $e^A$  (6)

b) Evaluate  $\int_0^\pi \frac{d\theta}{3 + 2\cos\theta}$  (6)

c) Find the singular value decomposition of  $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$  (8)

Q.3. a) Find the extremal of  $\int_0^\pi (y'^2 - y^2) dx$  given  $y(0) = 0, y(\pi) = 0$  (6)

b) Verify Cayley Hamilton theorem for  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$  and hence find  $A^{-1}$  &  $A^4$  (6)

c) Expand  $f(z) = \frac{1}{(z-1)(z-2)}$  in the regions (i)  $1 < |z-1| < 2$  (ii)  $|z| < 1$  (8)

Q.4. a) Construct an orthonormal basis of  $R^3$  using Gram Schmidt process to  $S = \{(3,1),(2,3)\}$  (6)

b) Find the extremum of  $\int_{x_0}^{x_1} (2xy + y''^2) dx$ . (6)

c) Reduce the quadratic form  $6x^2 + 3y^2 + 3z^2 - 4xy + 4xz - 2zy$  to canonical form and hence, find its rank, index and signature and value class. (8)

Q.5. a) Using Residue theorem evaluate  $\int_C \frac{z^2}{(z-1)^2(z+1)} dz$  where C is  $|z|=2$ . (6)

b) Find the linear transformation  $Y=AX$  which carries  $X_1 = (1, 0, 1)'$ ,  $X_2 = (1, -1, 1)'$ ,  $X_3 = (1, 2, -1)'$  onto  $Y_1 = (2, 3, -1)'$ ,  $Y_2 = (3, 0, -2)'$ ,  $Y_3 = (-2, 7, 1)'$  (6)

c) Check whether  $V = \mathbb{R}^2$  is a vector space with respect to the operations

$(x_1, 0) + (x_2, 0) = (x_1 + x_2, 0); k(x_1, 0) = (kx_1, 0)$  (8)

Q.6.a) Obtain Taylor's series expansion for  $f(x) = \frac{2z^3 + 1}{z(z+1)}$  about  $z = i$  (6)

b) Let  $W = span \left\{ (0, 1, 0), \left( \frac{-4}{5}, 0, \frac{3}{5} \right) \right\}$ , Express  $w = (1, 2, 3)$  in the form of  $w = w_1 + w_2$  where

$w_1 \in W$  &  $w_2 \in W^\perp$  (6)

c) Using Rayleigh- Ritz method, solve the boundary value problem  $I = \int_0^1 (2xy - y^2 - y'^2) dx$ ;

given  $y(0) = y(1) = 0$  (8)

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