

Time : 3 hrs

NB 1. Question No.I is compulsory

2. Attempt any three from the remaining six questions
3. Figures to the right indicate full marks

Q1a If Laplace transform of $erf(\sqrt{t}) = \frac{1}{s\sqrt{s+1}}$, then find $L\{e^t \cdot erf(2\sqrt{t})\}$ [20]

b Find the Orthogonal Trajectory of the family of curves given by $e^{-x} \cdot \cos y + x \cdot y = c$

c Find Complex Form of Fourier Series for e^{2x} ; $0 < x < 2$

d. If the two regression equations are $5x - 6y + 90 = 0$, $15x - 8y - 180 = 0$,

find the means of x and y , the Correlation Coefficient and Standard deviation of x if variance of Y is 1

Q2 Show that the function is Harmonic and find the Harmonic Conjugate $v = e^x \cdot \cos y + x^3 - 3xy^2$ [6]

b Find Laplace Transform of $f(t) = \begin{cases} t & ; 0 < t < 1 \\ 0 & ; 1 < t < 2 \end{cases}$, $f(t+2) = f(t)$ [6]

c. Find Fourier Series expansion of $f(x) = x - x^2$, $-1 < x < 1$ [8]

Q3 a Find the Analytic function $f(z) = u + iv$ if $v = \log(x^2 + y^2) + x - 2y$ [6]

b Find Inverse Z transform of $\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}$, $3 < |z| < 4$ [6]

c Solve the Differential Equation $\frac{d^2y}{dt^2} + 4y = f(t)$, $f(t) = H(t-2)$, $y(0) = 0$, $y'(0) = 1$ using Laplace Transform [8]

Q4 a Find $Z\{f(k) * g(k)\}$ if $f(k) = \left(\frac{1}{2}\right)^k$, $g(k) = \cos \pi k$ [6]

b Find the Spearman's Rank correlation coefficient between X and Y . [6]

| | | | | | | | | |
|---|----|----|----|----|----|----|----|----|
| X | 60 | 30 | 37 | 30 | 42 | 37 | 55 | 45 |
| Y | 50 | 25 | 33 | 27 | 40 | 33 | 50 | 42 |

c Find the inverse Laplace transform of i) $\frac{3s+1}{(s+1)^4}$ ii) $\frac{e^{4-3s}}{(s+4)^{5/2}}$ [8]

Q5 a Find Inverse Laplace Transform using Convolution theorem $\frac{1}{(s-4)^2(s+3)}$ [6]

b Show that the functions $f_1(x) = 1$, $f_2(x) = x$ are Orthogonal on $(-1,1)$. Determine the constants a, b such that the function $f(x) = -1 + ax + bx^2$ is Orthogonal to both $f_1(x), f_2(x)$ on the $(-1,1)$ [6]

c Find the Laplace transform of i) $e^{-3t} \int_0^t t \sin 4t dt$ ii) $\int_0^\infty \frac{e^{-t} - e^{-2t}}{t} dt$ [8]

Q6 a Fit a second degree parabola to the given data [6]

| | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|
| X | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| Y | 1.1 | 1.3 | 1.6 | 2 | 2.7 | 3.4 | 4.1 |

b Find the image of $\left|z - \frac{5}{2}\right| = \frac{1}{2}$ under the transformation $w = \frac{3-z}{z-2}$ [6]

c Find Half Range Cosine Series for $f(x) = x \sin x$ in $(0, \pi)$ and hence find $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots = \frac{\pi-2}{4}$ [8]
