

Time: 3 Hours

Marks: 80

- N.B.:1) Question no.1 is compulsory.
 2) Attempt any three questions from Q.2to Q.6.
 3) Figures to the right indicate full marks.

- Q1. a)** Find the Laplace transform of $e^{-t}t \cosh 2t$. [5]
- b)** Find the half-range cosine series for $f(x) = \begin{cases} 1 & , 0 < x < \frac{a}{2} \\ -1 & , \frac{a}{2} < x < a \end{cases}$ [5]
- c)** Find $\nabla \left(\bar{a} \cdot \nabla \frac{1}{r} \right)$ where \bar{a} is a constant vector. [5]
- d)** Show that the function $f(z) = z^3$ is analytic and find $f'(z)$ in terms of z . [5]
- Q2. a)** Find the inverse Z-transform of $F(z) = \frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}$, $3 < z < 4$. [6]
- b)** Find the analytic function whose imaginary part is $\tan^{-1} \left(\frac{y}{x} \right)$. [6]
- c)** Obtain Fourier series for the function $f(x) = \begin{cases} \frac{\pi}{2} + x & , -\pi < x < 0 \\ \frac{\pi}{2} - x & , 0 < x < \pi \end{cases}$, [8]
- Hence, deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ and $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$
- Q3. a)** Find $L^{-1} \left[\frac{s^2}{(s^2+1)(s^2+4)} \right]$ using convolution theorem. [6]
- b)** Show that the set of functions $\phi_n(x) = \sin \left(\frac{n\pi x}{l} \right)$, $n = 1, 2, 3 \dots$ is orthogonal in $[0, l]$. [6]
- c)** Using Green's theorem evaluate $\oint_C (e^{x^2} - xy)dx - (y^2 - ax)dy$ where C is the circle $x^2 + y^2 = a^2$. [8]
- Q4. a)** Find Laplace transform of $f(t) = \begin{cases} \frac{t}{a} & , 0 < t \leq a \\ \frac{(2a-t)}{a} & , a < t < 2a \end{cases}$ and $f(t) = f(t + 2a)$. [6]
- b)** Prove that a vector field \bar{f} is irrotational and hence find its scalar potential $\bar{f} = (y \sin z - \sin x) i + (x \sin z + 2yz)j + (xy \cos z + y^2)k$. [6]
- c)** Obtain the Fourier expansion of $f(x) = \left(\frac{\pi-x}{2} \right)^2$ in the interval $0 \leq x \leq 2\pi$ and $f(x + 2\pi) = f(x)$. Also deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ [8]
- Q5.a)** Use Gauss's Divergence Theorem to evaluate $\iint_S \bar{N} \cdot \bar{F} ds$ where $\bar{F} = 4xi + 3yj - 2zk$ and S is the surface bounded by $x=0, y=0, z=0$ and $2x+2y+z=4$. [6]
- b)** Find the Z-transform of $f(k) = ke^{-ak}$, $k \geq 0$. [6]
- c)** i) Find $L^{-1} \left[\frac{s+2}{s^2(s+3)} \right]$. [8]
 ii) Find $L^{-1} \left[\log \left(\frac{s+a}{s+b} \right) \right]$.
- Q6.a)** Solve using Laplace transform $(D^2 + 3D + 2)y = 2(t^2 + t + 1)$, with $y(0) = 2$ and $y'(0) = 0$. [6]
- b)** Find the bilinear transformation which maps the points $Z=1, i, -1$ onto the points $W=i, 0, -i$. [6]
- c)** Find Fourier sine integral of $f(x) = \begin{cases} x & , 0 < x < 1 \\ 2-x & , 1 < x < 2 \\ 0 & , x > 2 \end{cases}$ [8]