

(3hours)

[Total marks: 80]

- N.B.** 1) Question No. 1 is compulsory.
 2) Answer **any Three** from remaining
 3) Figures to the right indicate full marks

1. a) Verify Laplace equation for $u = \left(r + \frac{a^2}{r}\right) \cos \theta$. 5
- b) Find Laplace transform of $f(t) = e^{-3t} \sin 2t \cdot \cos 3t$. 5
- c) Obtain Fourier series for $f(x) = x$ in $(-\pi, \pi)$. 5
- d) Evaluate $\int_C (z^2 + 3z^{-4}) dz$ where C is the upper half of the unit circle from (1,0) to (-1,0). 5
2. a) Obtain the Taylor's and Laurent series which represent the function $f(z) = \frac{z}{(z+1)(z-2)}$ in the regions, i) $|z| < 1$ ii) $1 < |z| < 2$ 6
- b) Obtain Complex form of Fourier series for $f(x) = \cos hx$ in $(-\pi, \pi)$ 6
- c) Using Laplace transform, solve the differential equation:
 $\frac{dx}{dt} + 2x = \sin \omega t$, with $x(0) = 1$. 8
3. a) Solve $\frac{\partial^2 u}{\partial x^2} - 100 \frac{\partial u}{\partial t} = 0$ with $u(0, t) = 0, u(1, t) = 0, u(x, 0) = x(1 - x)$ taking $h = 0.1$ for three time steps up to $t = 1.5$ by Bender –Schmidt method. 6
- b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. 6
- c) Obtain half range Fourier sine series for $f(x) = \begin{cases} x, & 0 < x \leq \pi/2 \\ \pi - x, & \pi/2 \leq x < \pi \end{cases}$ 8
 Hence, prove that –

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

[TURN OVER]

Q. P. Code: 25571

4. a) Find the orthogonal trajectory of the family of curves $2x - x^3 + 3xy^2 = c$ 6
 b) Find the Fourier series for $f(x) = x|x|$ in $(-1, 1)$. 6

c) Find the inverse Laplace transform of:-

i) $F(s) = \frac{1}{s(s^2+16)}$, using Convolution theorem, ii) $F(s) = \cot^{-1}(s + 1)$. 8

5. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

$u(0, t) = 0, u(5, t) = 100, u(x, 0) = 20$ taking $h = 1$ for one-time step. 6

- b) Find the image of the circle $|z| = 4$ in the z -plane under the transformation $w = z + 4 + 3i$. Draw the sketch. 6

c) Find the analytic function $f(z) = u + iv$ if

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2 \cos x - e^y - e^{-y}}$$

when $f\left(\frac{\pi}{2}\right) = 0$. 8

6. a) Using Residue theorem, evaluate, $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos \theta}$ 6

- b) Using Laplace transform, evaluate $\int_0^{\infty} e^{-t}(1 + 3t + t^2)H(t - 2)dt$ 6

- c) A tightly stretched string with fixed end points $x = 0$ and $x = l$, in the shape defined by $y = kx(l - x)$ where k is a constant is released from this position of rest. Find $y(x, t)$, the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. 8