1T00514 - S.E.(CHEMICAL)(Sem IV) (CBSGS) / 38702 - APPLIED MATHEMATICS - IV

**(3 Hours)** **Marks : 80**

**N.B.**
1) **Question No. ONE is compulsory.**
2) **Solve any THREE Questions out of remaining FIVE.**
3) **Figures to the right indicate full marks.**
4) **Write the sub –questions of main question collectively together.**

**Q, 1.** a) Solve following PDE by the separation of variable method, **5**

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u, \text{ Given } u(x, 0) = 6e^{-3x}.$$

b) Evaluate $\int_A^B (y^2 dx + xy dy)$ *along* $y = 2t,\ x = t^2$ *where* $A(1, -2)$ *to* $B(0, 0)$. **5**

c) Obtain the Fourier expansion of $x^2$ in $(-\pi, \pi)$. **5**

d) Express the function $f(x) = \begin{cases} 1, & for\ |x| < 1 \\ 0, & for\ |x| > 1 \end{cases}$ as Fourier Transform. **5**

**Q. 2.** a) Find the Fourier series for $f(x) = 2x - x^2$, in (0, 3) whose period is 3. **6**

b) A rod of length L has its ends A & B kept at $0^0$C & $100^0$C resp. until steady state conditions prevail. If the temperature at A is raised to $25^0$C and that of B is reduced to $75^0$C & kept so , find the temperature u(x, t) at a distance x from A & at time t. **6**

c) Using Gauss's divergence theorem evaluate $\iint_S \overline{N}.\overline{F}$ ds where $\overline{F} = 2xi + xyj + zk$ over the region bounded by the cylinder $x^2 + y^2 = 4, z = 0, z = 6$. **8**

**Q. 3.** a) Obtain the complex form of Fourier series for f(x) = cosh2x + sinh2x in ( - 5, 5 ) **6**

b) Obtain half range cosine series for $f(x) = x\sin x$ in $(0, \pi)$. **6**

c) Evaluate by Green's Theorem in the plane for $\int_c (xy + y^2)dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ & $y = x^2$. **8**

**Q. 4.** a) Express the function $f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & x \ge \pi \end{cases}$, as Fourier sine integral. **6**

b) Show that the set of functions $\sin(\frac{\pi x}{2l})$, $\sin(\frac{3\pi x}{2l})$, $\sin(\frac{5\pi x}{2l})$,....... is orthogonal over $(0, l)$. **6**

**c)** A tightly stretched string with fixed end points $\boldsymbol{x = 0, x = l}$ in the shape defined by $\boldsymbol{y = kx(l - x)}$ where k is a constant is released from the position of rest. Find y. **8**

**Q. 5. a)** Obtain Fourier Series for the function f(x) = $\pi + x, \ -\pi \le x \le 0$
$= \pi - x, \quad 0 \le x \le \pi.$ **6**

**b)** Determine the solution of one dimensional equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary conditions $u(0, t) = 0, u(l, t) = 0, \ u(x, 0) = x, (0 < x < l), l$ being the length of the rod. **6**

**c)** Find the Fourier series for f(x) = $\begin{cases} 2, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$ **8**

**Q. 6. a)** Find Fourier sine Transform of $f(x) = e^{-ax}$. **6**

**b)** Using Stoke's Theorem evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = (x^2 + y^2)i + (x^2 - y^2)j$ and C is the is the boundary of the region enclosed by circles, $x^2 + y^2 = 4, x^2 + y^2 = 16$ . **6**

**c)** Find the Fourier series for f(x) = $sinax$ in [0 , 2π ]. **8**

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