1T00424 - S.E.(BIOTECHNOLOGY)(Sem IV) (Choice Based) / 40201 - APPLIED MATHEMATICS- IV

(3 hours) **Total Marks:80**

N.B: (1) Question no.**1** is **compulsory.**

(2)Attempt any **three** questions from remaining **five** questions.

(3)**Figures** to the **right** indicate **full** marks.

(4)Assume suitable data if necessary.

1. (a) Obtain the Fourier expansion of $f(x)=x^2,\ -l<x<l$. **(5)**

(b) Find the Fourier Transform of $f(x)=\begin{cases} e^{i\mu x}, & a<x<b \\ 0, & x<a, x>b \end{cases}$ **(5)**

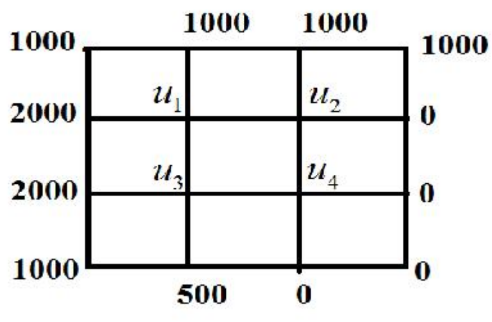
(c) Solve Partial differential equation $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ by method of separation of variables given $u(x,0)=4e^{-x}$. **(5)**

(d) Find the total work done in moving a particle in the force field $\overline{F}=3xyi-5zj+10xk$ along $x=t^2+1, y=2t^2, z=t^3$ from $t=1$ to $t=2$. **(5)**

2. (a) Prove that the functions $f_1(x)=1, f_2(x)=x, f_3(x)=\frac{3x^2-1}{2}$ are orthogonal over $(-1,1)$. **(6)**

(b) Express the fourier cosine integral representation of the function $f(x)=e^{-ax},\ x>0$ and hence, show that $\int_0^\infty \frac{\cos\mu x}{\mu^2+1}d\mu=\frac{®}{2}e^{-x}, x\geq 0$. **(6)**

(c) Solve Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for the following data by successive iterations (calculate first two iterations) **(8)**



3. (a) Find the Fourier sine transform of f(x) if $f(x)=\begin{cases} \sin kx, & 0\leq x<a \\ 0, & x>a \end{cases}$. **(6)**

(b)Using Green's Theorem evaluate $\int_C (e^{x^2} - xy)dx - (y^2 - ax)dy$ where C is the circle $x^2 + y^2 = a^2$. **(6)**

(c) Obtain fourier series for $f(x) = \begin{cases} x + \frac{®}{2} & -® < x < 0 \\ \frac{®}{2} - x & 0 < x < ® \end{cases}$ Hence, deduce $\frac{®^6}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ...$ **(8)**

4.(a) Obtain the complex form of Fourier Series for $f(x) = \cosh ax \; in(-l, l)$. **(6)**

(b) Use Stoke's Theorem to evaluate $\int_C \left(xy\,dx + xy^2\,dy \right)$ where C is the square in xy-plane with vertices (1,0), (0,1), (-1,0) and (0,-1). **(6)**

(c) A tightly stretched string with fixed end points $x = 0 \; and \; x = l$ in the shape defined by $y = kx(l - x)$ from where k is constant is released from this position of rest. Find $y(x,t)$, the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. **(8)**

5.(a) Find the Fourier cosine transform of f(x) if $f(x) = e^{-x^2}$. **(6)**

(b) Expand $f(x) = x \sin x$ in the interval $0 \le x \le 2®$. **(6)**

(c). Determine the solution of one-dimensional heat equation by the method of separation of variables

$$\begin{aligned} &\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \; 0 \le x \le l, t \ge 0 \\ &u = u(x,t) \\ &u(0,t) = u(l,t) = 0 \; for\, all\, t > 0 \\ &u(x,0) = \frac{100x}{l}, \; for\, all\, x > 0 \end{aligned}$$ **(8)**

6.(a)Use Gauss's Divergence Theorem to evaluate $\iint_S \overline{N}.\overline{F} ds \; where \; \overline{F} = 4xi - 2y^2 j + 3z^2 k$ and S is the surface of the cube bounded by $x^2 + y^2 + z^2 = a^2$, $z = 0, z = b$. **(6)**

(b) Prove that $\overline{F} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k$ is irrotational and find the scalar potential for $\overline{F}$ and evaluate $\int \overline{F}.d\overline{r}$ along the line joining the points $(1, 2, -4) \; and \; (3, 3, 2)$. **(6)**

(c) Find the half range sine series for $f(x) = lx - x^2$, $0 < x < l$. Hence, deduce that $\frac{®^6}{960} = \frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + ...$ **(8)**
