1T00414 - S.E.(BIOTECHNOLOGY)(Sem IV) (CBSGS) / 38602 - APPLIED MATHEMATICS - IV

Q.P. Code :23177

[Time: Three Hours] **[Marks:80]**

Please check whether you have got the right question paper.

N.B:
1. Question.No.1 is compulsory.
2. Attempt **any three** questions from the remaining.
3. Figure to the right indicate full marks.

Q.1 a) Find the Fourier sine transform of $e^{-x/2}$. 05

b) Evaluate $\int_c \bar{F}.\overline{dr}$ where $\bar{F} = yzi + zxj + xyk$ and C is the portion of the curve $\bar{r} = acosti + bsint\, j + ctk\ form\ \ (0, \frac{\pi}{2})$. 05

c) Find the half range cosine series of $f(x) = x, 0 < x < 2$ 05

d) Expand xcosx in $(-\pi, \pi)$ as the Fourier series. 05

Q.2 a) Evaluate by Green's theorem $\int_c (x^2 - y)dx - (2y^2 + y)dy$ where C is the boundary of the region bounded by $y = x^2\ \ and\ \ y = 4$ 06

b) Obtain the complex form of Fourier series for $f(x) = e^{-ax}$ in $(-\pi, \pi)$. 06

c) Solve the following one dimensional heat flow problem $\frac{\partial u}{\partial u} = k\ \frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod subject to the following conditions:- 08

i) u is finite for $t \to \infty$
ii) $\frac{\partial u}{\partial x} = 0$ for x= 0, x=0 for any time t
iii) u= 0 when $x = l$ for all values of t.
iv) $u = u_0$ when t= 0 for $0 < x < l$

Q.3 a) Show that the functions $\{\sin(2n+1)\,x\}; n = 1, 2, 3$ ... . Are orthogonal on$[0, \pi/2]$. Hence construct the orthonormal set of function. 06

b) Find the Fourier cosine integral representation of the function $f(x) = x^2\ (0, a)$. 06

c) Using Gauss Divergence Theorem, evaluate $\iint_s \bar{F}\ \overline{N}ds$ where $\bar{F} = xi + yj + z^2k$ and S is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane z = 1. 08

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Q.4 a) A tightly starched string within fixed end points x =0 and x = L is initially In equilibrium position. It is set vibrating by giving to each of its point's velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = V_0 \sin^3(\frac{\pi x}{L})$. Find y(x,t) 06

b) Find Fourier expansion of $f(x) = 2x - x^2,\ 0 < x < 3$ 06

c) Show that $\bar{F} = (y^2 \cos x + z^3)i + (2ysinx - 4\,)j - (3xz^2 + 2)k$ is irrotational and find the scalar potential for $\bar{F}$ and evaluate $\int \bar{F}.\overline{dr}$ along the curve joining the points (0, 1, -1) and $(\frac{\pi}{2}, -1, 2)$. 08

Q.5 a) Obtain the solution $\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ to satisfy the following conditions 06

i) $u \to 0$ as $y \to \infty$ for all x
ii) $u = 0$ if $x = 0$ for all y
iii) $u = 0$ if $x = L$ for all y
iv) $u = Lx - x^2$ if $y = 0$ for values of $0 < x < L$

b) Find the Fourier intergral representation of $e^{-|x|}$ 06

c) Evaluate $\int_c (\nabla \times \bar{F}).\overline{ds}$ where 08

$\bar{F} = (2y^2 + 3z^2 - x^2)i + (2z^2 + 3x^2 - y^2)j + (2x^2 + 3y^2 - z^2)k = 0$
Over the part of sphere $x^2 + y^2 + z^2 - 2ax + az = 0$ cut off by the plane $z = 0$.

Q.6 a) Evaluate $\int_c \bar{F}.\overline{dr}$ where $\bar{F} = (x^2 + y^2)i - 2xyj$ and C is the rectangle in xy plane Bounded by $y = 0,\ x = a,\ y = b,\ y = 0.$ 06

b) Find the Fourier for $f(x) = x + x^2 in\ (-2, 2)$. 06

c) Find the Fourier for f(x) in $(0,2\pi)$ where $f(x) = \left(\frac{\pi - x}{2}\right)^2,\ 0 \le x \le 2\pi$ Hence deduce that $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots ....$ 08

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