1T00323 - S.E.(BIOMEDICAL)(Sem III) (Choice Based) / 50501 - APPLIED MATHEMATICS III

Time: 3 Hours **Total Marks: 80**

N.B. 1) Question No. 1 is compulsory.

2) Attempt any three out of remaining five Questions.

3) Figure to right indicate full mark.

Q. 1 a)Find the constant a, b, c, d ,e if F(z) = $ax^3+bxy^2+3x^2+cy^2+x+i(dx^2y-2y3+exy+y)$ is Analytic. **(05)**

b) Find directional derivatives of $\emptyset = 4xy^2- 2yz^2$ at $[1, -2, , 2]$ in the direction of 2i + 3j + 5k **(05)**

c) Show that the set of functions Fn(x) = Cosnx , n = 1,2,3 ....is orthogonal over $[-\pi, \pi]$ Hence construct orthonormal set **(05)**

d) Find $L[e^{-2t}\ cos2t.cos4t\ ]$ **(05)**

Q.2 a) Prove that Field $\bar{F} = (z^2 + 2x + 3y)i + (3x + 2y + z)j + (y + 2xz)k$ is irrotational . Find scalarpotentional $\emptyset$ such that $\bar{F} = \nabla\emptyset$ **(06)**

b) obtain Fourier expansion of $F(x) = 4 - x^2$ in $[0, 2]$ **(06)**

c) Using Laplace transform Solve the differential equation **(08)**

$y'' + 2y' - 3y = \sin t$, Given y(0)=0, y '(0) = 0

Q.3 a) Prove that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{sinx}{x} - cosx\right\}$ **(06)**

b) Find Complex form of fourier series of $f(x) = e^{3x}$ in $[-\pi, \pi]$ **(06)**

c) i) Find $L^{-1}\left\{ Log\left(1 + \frac{a^2}{s^2}\right)\right\}$ ii) $L^{-1}\left\{\left(\frac{e^{-2s}}{s^2+8s+25}\right)\right\}$ **(08)**

Q.4 a)Find analytic function f(z) whose imaginary part is $e^x(x\sin y + y\cos y)=c$ **(06)**

b) Find $L^{-1}\left\{\frac{s}{(s^2+4)\,((s^2+16)}\right\}$ by convolution theorem **(06)**

c) Find Fourier expansion of $f(x) = 1+\frac{2x}{\pi}$ $-\pi \le x \le 0$ **(08)**

$= 1-\frac{2x}{\pi}$ $0 \le x \le \pi$ Hence deduce that

$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots\ldots\ldots$

Q.5 a) Find L[f(t)],Where f(t) =2t , $0< t < \pi$ **(06)**

$= \pi$- t , $\pi < t < 2\pi$ & f(t +2π)= f(t)

b) Find Bilinear transformation which maps the points ∞, i,0 of Z plane onto 0, i, ∞ of W plane . also find the fixed points **(06)**

c) Verify Green's Theorem for $\oint(x^2 - xy^2)dx + (y^2 - 2xy)dy$ Where C is square with vertices (0 , 0) (2, 0) (0, 2) , (2, 2) **(08)**

Q.6 a) Using Gauss divergence Theorem , evaluate $\iint \bar{n}.\bar{F}\ ds$ Where $\overline{F}$ = x^3i + y^3j + z^3k & S is the surface bounded by sphere $x^2 + y^2 + z^2 = a^2$ **(06)**

b) Find image of region bounded by x=0, y=0, x = 1, y=1 in Z plane onto W plane under W = z + (2+ i) **(06)**

c) Find Fourier cosine integral representation of f(x) = x 0≤x≤1 **(08)**

= 2 – x 1≤x≤2

= 0 x> 2

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